SCORE: _____ / 20 POINTS +1

1. NO CALCULATORS ALLOWED ON THIS PART
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

Evaluate the following limits. Write "DNE" if a limit does not exist. SCORE: _____ / 8 PTS

You do not need to show the use of the limit laws. However, it must be clear how you got your answers.

[a] $\lim_{m \to 2} \dfrac{m^2 - 3m - 2}{2m^2 - 5m - 2}$

$= \dfrac{(2)^2 - 3(2) - 2}{2(2)^2 - 5(2) - 2}$

$= \dfrac{4 - 6 - 2}{8 - 10 - 2}$

$= \dfrac{-4}{-4} = \boxed{1}$

½ ½

[b] $\lim_{p \to -1} \dfrac{5 + 5p}{\sqrt{2p+3} + p} \cdot \left(\dfrac{\sqrt{2p+3} - p}{\sqrt{2p+3} - p} \right)$

$= \lim_{p \to -1} \dfrac{(5+5p)(\sqrt{2p+3} - p)}{2p + 3 - p^2}$ *

$= \lim_{p \to -1} \dfrac{5\cancel{(p+1)}(\sqrt{2p+3} - p)}{-(p-3)\cancel{(p+1)}}$ ½

$= \lim_{p \to -1} \dfrac{5(\sqrt{2p+3} - p)}{3 - p}$

$\Rightarrow \dfrac{5(1+1)}{3+1} = \dfrac{5(2)}{4} = \dfrac{10}{4}$

$\boxed{= \dfrac{5}{2}}$ ½

[c] $\lim_{a \to 3} \dfrac{a^2 - 9}{\dfrac{5}{a+2} - \dfrac{2}{a-1}}$

$= \lim_{a \to 3} \dfrac{a^2 - 9}{\left(\dfrac{5(a-1) - 2(a+2)}{(a+2)(a-1)} \right)}$ *

$= \lim_{a \to 3} \dfrac{a^2 - 9}{\left(\dfrac{3a - 9}{(a+2)(a-1)} \right)} = \lim_{x \to 3} \dfrac{(a^2 - 9)(a+2)(a-1)}{3(a-3)}$

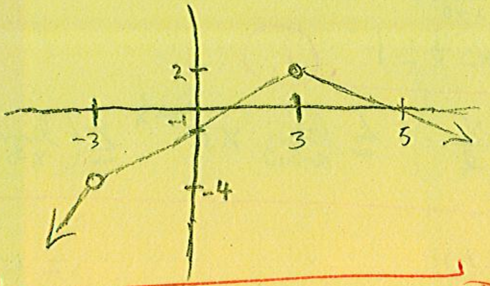
$= \lim_{a \to 3} \dfrac{(a+3)\cancel{(a-3)}(a+2)(a-1)}{3\cancel{(a-3)}}$ ½

$= \lim_{a \to 3} \dfrac{(a+3)(a+2)(a-1)}{3}$

$\Rightarrow \dfrac{(6)(5)(2)}{3} = \dfrac{60}{3}$

$\boxed{= 20}$ ½

[d] $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} 2x + 2, & \text{if } x < -3 \\ x - 1, & \text{if } -3 < x < 3 \\ 5 - x, & \text{if } x > 3 \end{cases}$



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$\boxed{= 2}$ ½

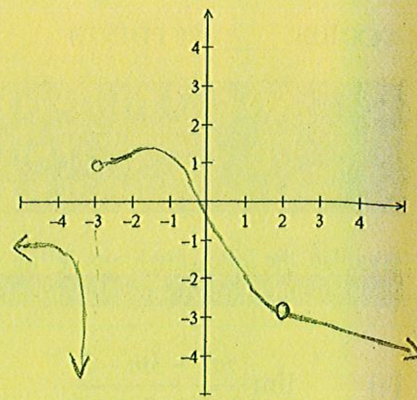
Sketch the graph of an example of a function that satisfies all the following conditions. **SCORE:** 2 **/ 2 PTS**

$\lim_{x\to -3^+} f(x) = 1$

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to 2} f(x) = -3$

$f(2)$ does not exist



The graph of f is shown on the right. Evaluate the following limits. Write "DNE" if a limit does not exist. **SCORE:** _____ **/ 3 PTS**

[a] $\lim_{x\to 1} \frac{7x}{5 - f(x)}$ **Show the proper use of limit laws to find your anwer.**

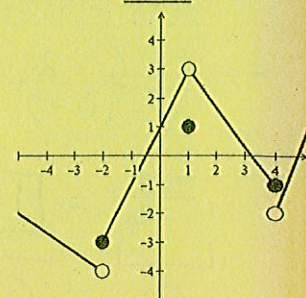
$$= \frac{\left(\lim_{x\to1} 7x\right)}{\left(\lim_{x\to1} 5\right) - \left(\lim_{x\to1} f(x)\right)}$$ ✱

$$= \frac{7(1)}{5-(3)}$$

$$= \frac{7}{5-3} = \boxed{\frac{7}{2}}$$ 1

[b] $\lim_{x\to -2^-} f(x)$

$\boxed{-4}$ 1



Prove that $\lim_{x\to 0} xe^{\cos\frac{1}{x}} = 0$. **SCORE:** _____ **/ 4 PTS**

$-1 \le \cos\frac{1}{x} \le 1$ 1

$$\left(\lim_{x\to0} \frac{x}{e}\right) \le \lim_{x\to0} xe^{\cos\frac{1}{x}} \le \left(\lim_{x\to0} xe\right)$$ ✱

$= \frac{0}{e} = 0$ $\qquad$ $= (0)(e) = 0$

$$0 \le \lim_{x\to0} xe^{\cos\frac{1}{x}} \le 0$$ ✱

Therefore, by the Squeeze theorem, $\lim_{x\to0} xe^{\cos\frac{1}{x}} = 0$

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